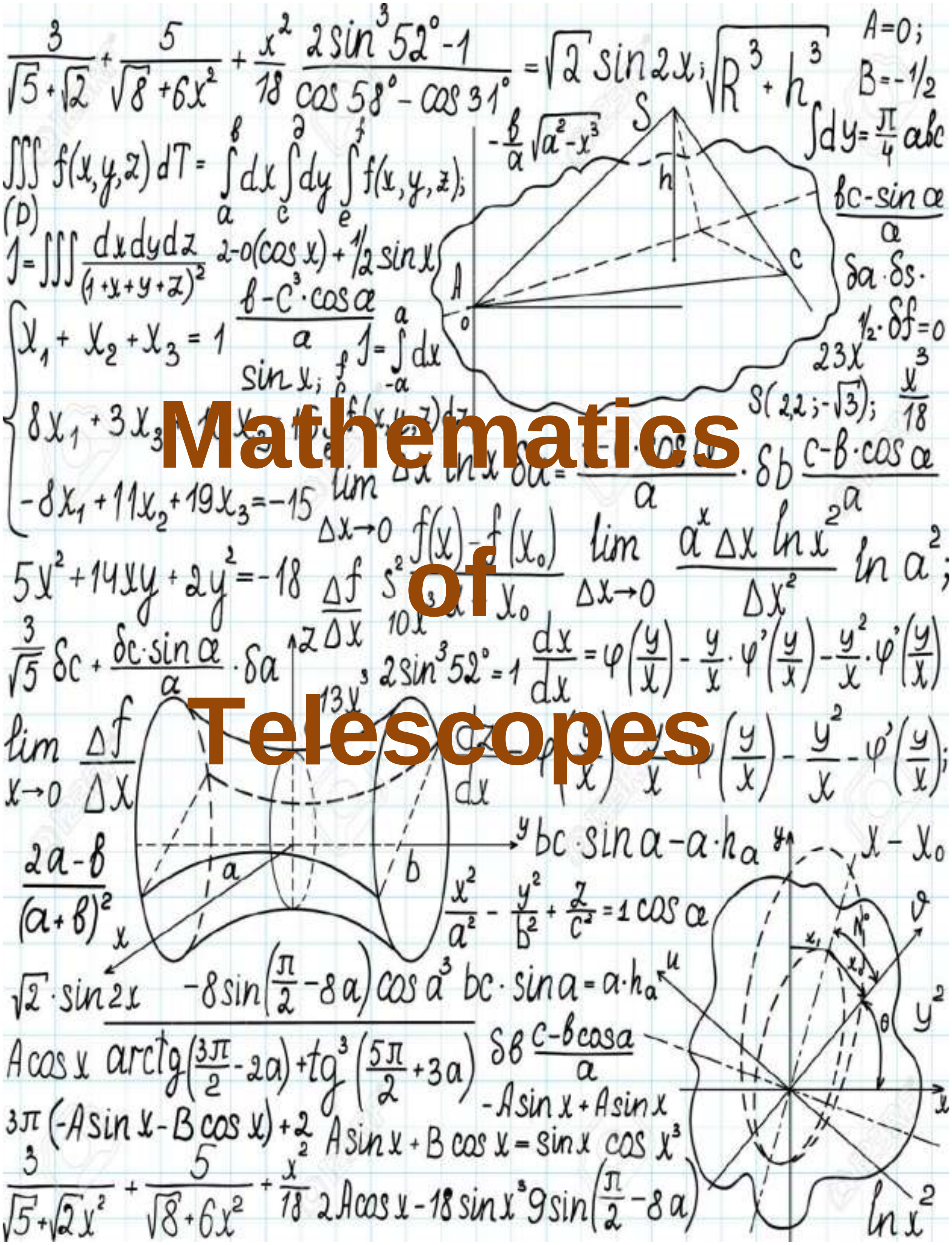
Mathematics of Telescopes



****All results are assuming a telescope with:

Aperture (D_O) = 200mm
Focal Length (f_O) = 1950mm
Focal Ratio (f_R) = 9.75

Introduction

So why telescope mathematics? On an everyday basis you probably do not need to understand the mathematics of your present or future telescope. But, if you want to know what the longest or shortest focal length eyepiece you should buy for your telescope is, there are a few things you should know about your telescope and mathematics will tell you what you need to know. Also, if you want to know if the large nebulae or star clusters you want to observe will fit into the field of view of your telescope, mathematics will tell you. Do you want to observe planets and the moon or deep sky Messier objects? Mathematics will help you decide on the telescope you should buy.

The following equations will help you answer the above questions and more. Each topic/equation begins on a new page. After each topic heading I have done the calculations using the specifications of the telescope with the assumed parameters given at the beginning of this article. And after each topic I have appended the definitions of each telescope parameter shown in the equations.

Magnification: 28X to 200X

Magnification is the ratio of the focal length of the telescope objective to the focal length of the eyepiece. Although astronomers do not usually discuss the magnification of their telescope, preferring to speak of the aperture, it can be handy to know this parameter in order to calculate other factors.

$$M = f_O / f_e$$

By rearranging the formula we can determine the focal length of the objective or the focal length of eyepiece if we know the magnification and either one of the other parameters.

$$f_O = M \times f_e$$

$$f_e = f_O / M$$

Using the assumed telescope parameters and an eyepiece with a focal length of 15mm, we can calculate the magnification of the telescope. Remember – the magnification will change with the focal length of the eyepiece.

$$M = 1950 / 15$$
$$= 130X$$

Thus, with a 15mm eyepiece, the telescope will have a magnification of 130X.

M = Magnification
f_O = Focal length of the objective lens/mirror
f_e = Focal length of the eyepiece

Focal Ratio: 9.75

This is the speed of the telescope. It determines the amount of light the telescope lets in and thus the dimmest objects you can see with the telescope. This is a ratio of the focal length of the telescope's objective to its aperture.

$$f_R = f_O / D_O$$

Again, by rearranging the formula we can determine the focal length of the objective or the diameter of the objective if we know the focal ratio and either one of the other parameters.

$$f_O = f_R \times D_O$$

$$D_O = f_O / f_R$$

Using the assumed telescope:

$$f_R = 1950 / 200$$
$$= 9.75$$

The focal ratio of the assumed telescope is f9.75.

f_R = Focal ratio
f_O = focal length of the objective
D_O = Diameter of Objective (Aperture)

Field of View (FOV)

The Field of View of a given telescope is dependent on the Field of View of the eyepiece being used and the magnification of the telescope while using that eyepiece. The field of view will determine how large an object you will be able to view using that eyepiece

Typically, an eyepiece has a field of view of 50-60°, although there are wide-field eyepieces that go up to as much as 100°.

To find what the field of view will be in your telescope, first find the field of view of the eyepiece from its specifications, then divide by the magnification of your scope (with that eyepiece). Expressed as an equation this comes out to be:

$$FOV_{scope} = FOV_e / M$$

FOV_{scope} = Field of View of the telescope
FOV_e = Field of View of the eyepiece
M = magnification

For example, using an eyepiece that has a 32mm focal length and a field of view of 70^o, in the assumed telescope, are you able to see the whole of the Orion Nebula (M42), which covers approximately 1.1^o of sky?

First calculate the magnification of my telescope using this lens:

$M = f_O / f_e$
$=1950 / 32$
$= 60.9X$ or $61X$

Then calculate the FOV of the assumed telescope using this lens:

$FOV_{scope} = FOV_e / M$
$= 70 / 61$
$=1.17^o$

The answer is YES. Just barely!

Dawes Limit

Before we go any further I need to introduce you to William Rutter Dawes since the next few formula require that you know what he is famous for in astronomy.

In 1867, William Rutter Dawes determined the practical limit on resolving power for a telescope. This is known as the Dawes Limit. Dawes defined this as “the closest that two stars could be together in the sky and still be seen as two stars”. The Dawes Limit is **4.56 seconds of arc** divided by the diameter of the objective **in inches**. Since telescopes objectives are usually measured in millimeters (mm), we can multiply **4.56 seconds of arc** by 25.4 to convert to the metric system (in millimeters), which gets you to 115.8 seconds of arc divided by the diameter of the objective in millimeters (mm). You then round this up to 120 seconds of arc, a number that is more convenient when doing the math in your head.

Resolving Power (Dawes Limit): 0.6 arc sec

From the study that Dawes did we know the human eye can resolve two objects that have 2 arc-minutes of separation center to center and are each 1 arc-minute in size (see diagram in next topic). Because we measure resolving power in arc-seconds, 2 arc-minutes is equal to 120 arc-seconds.

$$P_R = 120 / D_O$$

Using the assumed telescope, calculating the resolving power gives us:

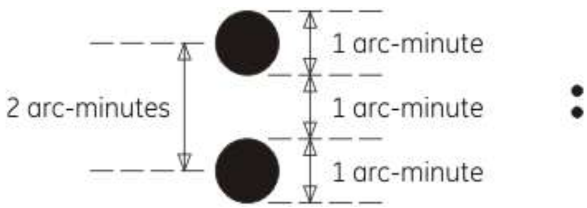
$$= 120 / 200$$
$$= 0.6 \text{ arc sec}$$

This means that the assumed telescope can resolve two objects that are 0.6 arc seconds apart as viewed by the naked eye.

P_R = Resolving Power in arc-seconds
D_O = Diameter of Objective

Maximum Magnification: 200X

It has been determined that a person with 20/20 vision can distinguish a feature that has a size of one minute of arc. (There are 60 arc-minutes to a degree, and 60 arc-seconds in an arc-minute.) And in order for a person to distinguish two stars as separate, the stars need to be separated by 2 minutes of arc, center to center -- see the picture below.



Thus, for a person to see two objects that have a separation of less than 2 minutes of arc, a telescope needs to magnify the separation to one the eye can resolve, which is 2 minutes of arc, or 120 arc-seconds. So then we have:

$$M_{max} \times P_R = 120$$
$$M_{max} \times 120/D_O = 120$$
$$M_{max} = D_O$$

Notice that the maximum magnification of any telescope can be determined by simply checking the telescope's aperture. Most telescopes are capable of about **50% higher** than the scope's maximum capability as determined by the formula but most manufactures double the calculated value. So, be wary of the stated maximum magnification.

The maximum magnification for our assumed telescope is 200X since the objective diameter is 200mm. But adding another 50% (100X) gives us 300X. Maximum magnification is also limited by atmospheric conditions. So be careful when quoting maximum magnification.

M = Magnification
P_R = Resolving Power in arc-seconds
D_O = Diameter of Objective Lens (Aperture) in millimeters
M_{max} = Maximum Magnification

Eyepiece focal length for maximum magnification

From the equation for magnification we have

$$M = f_O / f_e$$

And we want the value of $\mathbf{f_{e\text{-}min}}$ to get us to $\mathbf{M_{max} = D_O}$

So since $\mathbf{D_O = f_O / f_{e\text{-}min}}$

Therefore $\mathbf{f_{e\text{-}min} = f_O / D_O}$.

Since the f-ratio $\mathbf{f_O / D_O = f_R}$ we then have, quite simply:

$$f_{e\text{-}min} = f_R$$

So, the eyepiece focal length to get the maximum magnification can be found by just looking at the f-ratio for the scope! If I look at the front of a telescope and it says "f/9.5", I know the smallest eyepiece I can use with that scope is a 9.5mm eyepiece.

Remember, however, that the telescope is capable of 50% more magnification. So, for our assumed telescope:

$$f_{e\text{-}min} = f_o / M \text{ or } 1950/300 = 6.5$$

Thus the focal length of the eyepiece for maximum magnification with our assumed telescope is 6.5mm.

> M = Magnification
> M_{max} = Maximum magnification
> f_O = Objective focal length
> f_e = Eyepiece focal length
> $f_{e\text{-}min}$ = Eyepiece minimum focal length
> D_O = Objective diameter
> f_R = Focal ratio

Minimum Magnification: 28X

The minimum magnification of a telescope is dependent on the maximum diameter of the pupil in the human eye (D_{ep}). Therefore, using the formula for magnification derived for the diameters we get:

$$M_{min} = D_o / D_{ep}$$

Since the maximum exit pupil diameter (D_{ep}) is 7mm* (the maximum diameter of the pupil of the human eye) then:

$$M_{min} = D_o / 7$$

Using the assumed telescope parameters the minimum magnification is:

$$M_{min} = 200 / 7$$
$$= 28X$$

The equation for the maximum focal length for the eyepiece required to give this minimum magnification is the same as for the equation for the minimum focal length eyepiece for the maximum magnification.

$$f_{e\text{-}max} = f_o / M$$
$$= 1950/28 = 70$$

Therefore the maximum eyepiece focal length for minimum magnification with our assumed telescope is 70mm. This is not a practical size for an eyepiece since the largest eyepieces are usually in the 40mm range. Thus the minimum magnification for this telescope should be about **48X**.

*Remember, the maximum diameter of the pupil diminishes with age.

M_{min} = Minimum magnification
D_O = Objective Diameter
D_{ep} = Diameter of the pupil of the eye (7mm)
$f_{e\text{-}max}$ = Maximum eyepiece focal length

Gathering Power (Light Grasp / Brightness Increase): 816X

The Gathering Power of a telescope refers to how much more light can the telescope gather compared to the human eye and is expressed as the ratio of the area of the objective to the area of the human eye pupil.

$$G_L = (D_O / D_{eye})^2$$

Using our assumed telescope, the Light Grasp is calculated to be:

$$\begin{aligned} G_L &= (200 / 7)^2 \\ &= (28.57)^2 \\ &= 816 \end{aligned}$$

So, our assumed telescope gathers 816X as much light as the human eye.

Some observers say we should quote this value instead of magnification when asked how powerful a telescope is.

G_L = Light Grasp
D_O = Objective Diameter
D_{eye} = Diameter of the Eye (maximum 7mm)

Limiting Magnitude: 13.5

The limiting magnitude is the dimmest object that can be discerned using a particular telescope and it depends on the telescope's objective diameter.

The brightness increase is calculated as:

$$G_L = (D_O/D_{eye})^2$$

The brightness increase in terms of magnitudes is 2.5 times the logarithm of G_L or

$$2.5 \times \log(G_L)$$

so it's just

$$G_{mag} = 2.5 \times \log((D_O / D_{eye})^2)$$

Since $\log(x^2) = 2 \times \log(x)$. So then:

$$\begin{aligned} G_{mag} &= 2.5 \text{ x } \log((D_O / D_{eye})^2) \\ &= 2.5 \text{ x } 2 \text{ x } \log(D_O / D_{eye}) \\ &= 5 \text{ x } \log(D_O / D_{eye}) \end{aligned}$$

When you divide two numbers you subtract their logarithms, so subtracting the log of D_{eye} from the log of D_O , then substituting 7mm for D_{eye} , we get:

$$\begin{aligned} G_{mag} &= 5 \text{ x } \log(\log(D_O) - \log(D_{eye})) \\ &= 5 \text{ x } \log(D_O) - 5 \text{ x } \log(7) \end{aligned}$$

Since log(7) is about 0.8, then 5×0.8 = 4 so our equation for the increase in star magnitude is:

$$G_{mag} = 5 \text{ x } \log(D_O) - 4$$

This represents how many more magnitudes the scope lets me see, over and above what my eyes alone can see. Then to find the faintest magnitude I can see in the scope, we simply add Gmag to the faintest magnitude our eye can see, magnitude 6. This is the magnitude limit of the scope, Lmag:

$$L_{mag} = G_{mag} + 6 = 5 \times \log(D_O) - 4 + 6$$

This simplifies down to our final equation for the magnitude limit, L_{mag}, of the scope:

$$L_{mag} = 2 + 5 \times \log(D_O)$$

Did you notice that the magnitude limit of the telescope — the faintest object you can see in the telescope — depends **only** on the diameter of the objective?

Using the above equation the limiting magnitude for our assumed telescope is:

$$L_{mag} = 2 + 5 \times \log(200)$$
$$= 2 + (5 \times 2.3)$$
$$= 13.5$$

Thus with this telescope we should be able to view objects with a magnitude down to13.5.

G_L = Brightness Increase
G_{mag} = Brightness Increase in magnitudes
D_O = Objective Diameter
D_{eye} = Diameter of the Eye (maximum 7mm)
L_{mag} = Limiting Magnitude

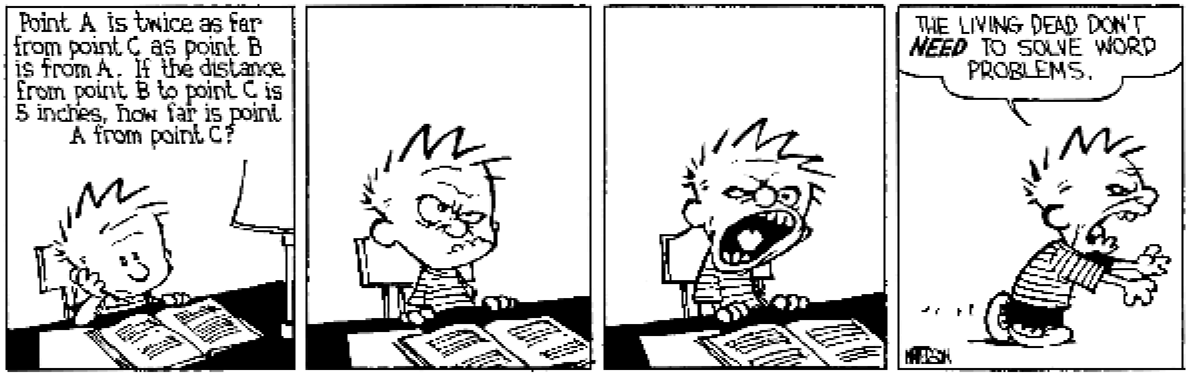
Focal Ratio Comparison

Large/Slow focal ratios (e.g.: f11-f15) - longer focal length and smaller aperture = higher magnification, less light gathering. Best suited to higher power lunar, planetary, and binary star observing and high power photography.

Small/Fast focal ratios (e.g.: f4-f5) – shorter focal length and larger aperture = lower magnification, good light gathering. Best for lower power wide field observing and deep space photography.

Medium focal ratios (e.g.: f6-f10) - Work well for all applications.

Our assumed telescope has a focal ratio of f9.75. So it falls into the **medium focal ratio** range.



For more information and more equations go to:

http://www.rocketmime.com/astronomy/Telescope/telescope_eqn.html

A lot of the information given here came from these pages.